Structure of Stark Broadened H$_\beta$ Lines at low Electron Densities

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The profile of the hydrogen H$_\beta$ line emitted by a hydrogen low-temperature afterglow plasma has been measured at electron densities $1.8 \cdot 10^{15}$, $1 \cdot 10^{15}$ and $5 \cdot 10^{14}\ \mathrm{cm}^{-3}$ by means of a 10-channel analyzer system with a spectral resolution of 0.1 Å. All profiles showed weak plasma satellites suddenly appearing at wavelength distances which correspond to the electron plasma frequency and showing "satellite line wings" which slowly smooth out at larger wavelength distances. In the line core—where all quasi-static and ion dynamic theories predict a minimum—appeared an intensity maximum well separated from the closely spaced intensity shoulders. This new feature not yet predicted by line broadening theories and not yet observed by other experimentalists is probably due to low-frequency ion oscillations which couple with the excited atomic system. Such an effect would explain discrepancies between available Stark broadening theories and the mass-dependent structure of line profiles recently observed in other experiments. The effect may possibly be useful for measuring collective low-frequency oscillations in plasmas under very special conditions.

Introduction; Incompatibility of Experiments with Theory

In this paper we describe experimental results obtained for H$_\beta$ lines which were emitted from an afterglow plasma at relatively low electron densities and temperatures. The work was motivated by recently observed discrepancies between experiments and theory for some prominent spectral lines of atomic hydrogen and helium. The theoretical profiles of the He I lines $\lambda = 4471$ Å ($2^3\mathrm{P} - 4^3\mathrm{D}$) and $\lambda = 4470$ Å ($2^3\mathrm{P} - 4^3\mathrm{F}$) calculated by Griem [1] and by Barnard, Cooper and Shamey [2] were not confirmed by Burgess and Cairns [3] and Jenkins and Burgess [4]. Burgess [5] pointed out that these discrepancies could be due to ion dynamics not taken into account in the theory which treats the ions as quasi-static. New measurements of Drawin and Ramette [6] and Diatta, Czernichowski and Chapelle [7] did not confirm the results of Burgess and Cairns and Jenkins, there were on the contrary some indications that the mentioned theoretical profiles fitted the observed ones relatively well. At high electron densities Greig, Jones and Lee [8] also confirmed agreement with the quasi-static theory.

In the meantime new theoretical profiles of the He I lines $\lambda = 4471$ Å and $\lambda = 4470$ Å were published by Lee [9] and Barnard, Cooper and Smith [10] now including dynamic ion broadening. These authors found an appreciable modification of the line profiles for electron densities $N_\mathrm{e} < 5 \cdot 10^{15}\ \mathrm{cm}^{-3}$ compared to the purely quasi-static theories developed so far. Their results agreed well with the earlier measurements of Burgess and Cairns [3] and also with more recent experiments made by Barnard and Stevenson [11]. For electron-ion densities greater than $5 \cdot 10^{15}\ \mathrm{cm}^{-3}$, Barnard, Cooper and Smith [10] found the influence of ion dynamics to be negligible so that the quasi-static theory should adequately describe the profiles. Their results are in contradiction to most recent calculations of Segré and Voslamber [12] and Brissaud et alias [13]. The papers of Segré and Voslamber and of Brissaud et al. confirmed for the line centers the experimental findings of Drawin and Ramette, but for the valley between the two lines the two papers give contradictory results and only the calculations of Segré and Voslamber agree with the experiments. The important result of the theoretical investigations of Brissaud et al. is that ion dynamics seems to have an effect which is just opposite to the observations of Burgess and Cairns and the calculation of Lee and of Barnard, Cooper, Smith, i. e. that it should lead to a lowering of the valley intensity and to an increase of the central part of the forbidden line at $\lambda = 4470$ Å.

Parallel to this, measurements and calculations were made for hydrogen lines. In 1973, Kelleher and Wiese [14] reported about ion perturber dependent structure of the H$_\beta$ line emitted by a cascade arc at electron densities of the order of $5 \cdot 10^{16}$ to

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$1 \cdot 10^{17}\ cm^{-3}$. These authors found for given electron density and temperature a change in the central structure of the H_β line which scaled with the inverse square root of the reduced mass of the radiator-ion perturber system. The measurements were extended to the four lines H_α, H_β, H_γ, H_δ by Wiese, Kelleher and Helbig [15] who confirmed for all profiles a dependence on the reduced mass. Extrapolation of the results for H_α and H_β to infinite reduced mass (i. e. quasi-static ion perturbers) seems to indicate agreement with theoretical calculations of Kepple and Griem [16] and Vidal, Cooper and Smith [17] who assumed the ions to be quasi-static. A change of the central structure of H_β with ion perturber mass was also observed by Burgess and Mahon [18]. The general trend of the central structure of H_β with the type of ionic perturber could be confirmed by calculations of Lee [19] and Cooper, Smith and Vidal [20], but they are in contradiction to theoretical results of Capes, Stamm and Voslamber [21] and Stamm and Voslamber [22]. At present appreciable quantitative discrepancies between experiment and theory persist which cannot be attributed to experimental uncertainties provided that plasma inhomogeneities can be excluded as a possible source of error (this point is discussed later).

The measurements of the Wiese group were made on a wall-stabilized cascade arc, those of the Burgess group on a Z-pinch produced afterglow plasma. Quite a different light source was used by Hey and Griem [23] who measured the line profiles of H_α, H_β, D_β and of He I $\lambda = 4471$ Å, $\lambda = 4470$ Å. The measured central structure of H_α agreed rather well with the quasi-static theory of Kepple and Griem [16] and deviated strongly from the "unified theory" results of Vidal et al. [17]. On the other hand, the experimental structure of the hydrogen H_β and deuterium D_β lines deviated in the line center from both the Kepple-Griem and the Vidal et al. calculations. These characteristic deviations from the theory were explained as being due to plasma inhomogeneities, ion dynamics as a possible reason for the observed discrepancies was considered as highly improbable. As plasma inhomogeneities can indeed lead to incontrollable alteration of the line structure we have thoroughly checked this point in our experiment. In anticipating the conclusion of our investigations: it is highly improbable that the observed characteristic discrepancies between theory and experiments are due to plasma inhomogeneities (see later).

A New Experimental Concept in Ion Dynamic Broadening

At present the situation is such that the theoretical and experimental profiles of a large number of spectral lines seem to agree relatively well in the region of the (far) line wings, while in the central part experiments and theory are contradictory. The observed discrepancies between theory and experiment seem to be much larger than experimental uncertainties. In the special case of H_β different experiments have shown that the intensity in the dip between the two adjacent shoulders depends on the mass of the most abundant ionic perturber. Further, it is also an experiment fact that a number of experimental profiles published in the literature show sometimes strange irregularities either in the line center or in the line wings, or in both regions. Inspection of the many experimental hydrogen line profiles published so far seem sometimes to indicate for H_α, H_β and H_γ a very small but non negligible bump corresponding to the electron plasma frequency. Peaks at this frequency distance have often been observed in highly turbulent plasmas. (A detailed discussion can be found in Griem's book [24]). However, rough estimates seem to indicate that the turbulence level is too low in plasmas discussed in the present context and that excitation of electron plasma waves can be excluded as a possible reason for the discrepancies between experiments and theory. We think that this latter point demands further detailed investigations. It might be that due to the final dimensions of all laboratory plasmas the Fourier coefficients of special wave modes become especially large. The influence of finite dimensions is twofold: First, finite dimensions impose boundary conditions to the wave equation; second: they increase deviations from thermodynamic equilibrium. Both effects can principally lead to an increase of the excitation level of both electron and of special ion wave modes. Special attention should be given to low-frequency waves excited at and near the ion plasma frequency. As the ion plasma frequency will lead to satellites near to the line center, a collective ion effect might explain the mass dependent feature in the line center.

It follows from this that the search for "ion dynamic broadening" effects in spectral lines should be based on a new concept, namely to search for features which could be related to low-frequency plasma wave effects. Special attention should be paid to ion waves the manifestations of which should be visible in the line center. As far as the hydrogen lines are concerned, the best candidate for such measurements is the H_β line which has a more or less pronounced minimum in the line center. In the frame of existing line broadening theories the minimum is deepest at low electron densities. One should therefore measure the line profile at electron densities sufficiently low in order to be sensitive towards ion wave features – if they exist – and at the same time at electron densities sufficiently high in order to have a good wavelength separation from the two adjacent intensity maxima. Moreover, the spectral resolution must be sufficiently high and Doppler effect should be kept at a very low level. Inspection of the theoretical line profiles of the H_β line leads to the conclusion that the optimum conditions for such measurements lie somewhere between $5 \cdot 10^{14}\,\mathrm{cm}^{-3}$ and $5 \cdot 10^{15}\,\mathrm{cm}^{-3}$ for the electron density. The gas temperature should lie below 10^4 K. These conditions can be fulfilled in afterglow plasmas.

Experimental

The experimental device was quite similar to the one used for the measurement of the He I line $\lambda = 4471$ Å, $\lambda = 4470$ Å (see Ref. [6]) with the only difference that we have reduced the effective plasma length to 22 cm and the tube diameter to 2.4 cm. The discharge tube is schematically shown in Figure 1.

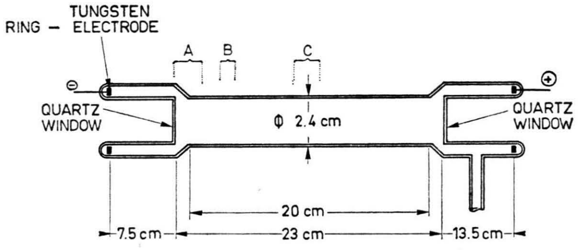

Fig. 1. Schematic representation of the discharge tube; side-on measurements were made in the regions A, B and C.

The plasma was built up by a slowly increasing current which was short-circuited after 1 msec. Immediately before short-circuiting the plasma current had reached 1 KA. 40 μsec after short-circuiting the current was definitely zero.

The filling pressure (pure hydrogen gas) was 3 torr for all measurements. The measurements were performed during the afterglow in the temperature range 5000 K to 3500 K (see Fig. 1 in Ref. [6]). A He-Ne laser interferometer served for measuring the electron density. A special 10-channel analyzer system already described in Ref. [6] served for measuring the spectral intensities continuously as a function of time simultaneously for the ten channels. Composition of the whole line profile was as described in Reference [6]. We mention that one channel corresponds to a spectral width of 0.1 Å. The ten channels differ from this value by not more than 2%. The dispersion was measured by means of an iron arc which emits in the H_β region several well-separated lines of known wavelengths. The ten-channel measuring head was mounted on a mouvable support the lateral position of which could be controlled with a precision better than 0.005 mm.

The line profile measurements were performed in axial direction (end-on observation) and perpendicular to the discharge tube (side-on observation) in three different regions (region A, B and C of Figure 1). Special attention was paid to the side-on measurements in "region A" which must be considered as a region of inhomogeneity in the case of end-on measurements. Measurements were performed in different planes parallel to the quartz window. End-on and side-on measurements led to consistent results for the line profiles, expecially for the feature in the line center (see later). Plasma inhomogeneities as a possible source of error must therefore be excluded. In the following we describe the experimental results as obtained by the end-on measurements, since they do not contain the uncertainties associated with the deconvolution procedure by Abel inversion.

Results

The Figs. 2 to 4 show measured H_β line profiles (solid curves) for three different electron densities

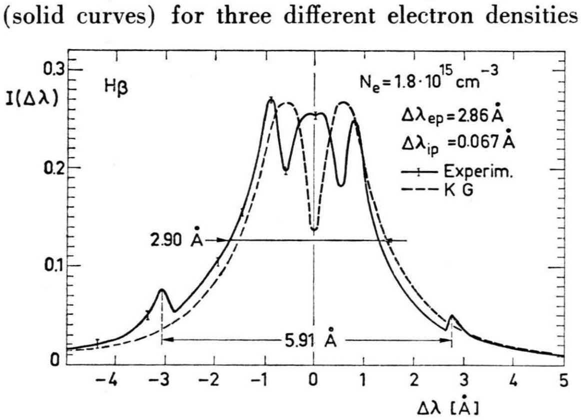

Fig. 2. H_β line profile for measured electron density $N_e = 1.8 \cdot 10^{15}\,\mathrm{cm}^{-3} \pm 7\%$. ——— experiments, – – – theory (Kepple and Griem).

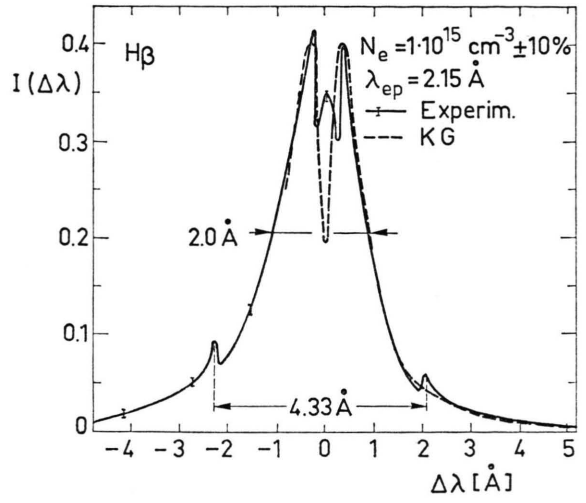

Fig. 3. H_β line profile for measured electron density $N_e = 1.0 \cdot 10^{15}\ \mathrm{cm}^{-3} \pm 10\%$. ——— experiment, —·—·— theory (Kepple and Griem).

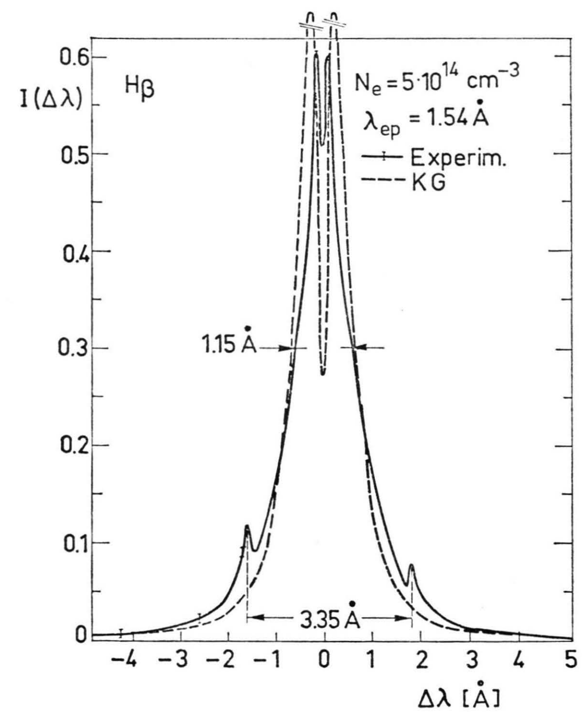

Fig. 4. H_β line profile for measured electron density $N_e = 5 \cdot 10^{14}\ \mathrm{cm}^{-3} \pm 20\%$, —— experiment, - - - theory (Kepple and Griem).

N_e. All lines are normalized to unity according to

$$\int_{H_\beta \text{ line}} I(\Delta\lambda)\, \mathrm{d}(\Delta\lambda) = 1 .$$

Broken curves represent theoretical profiles (quasi-static theory) after Kepple and Griem [16], see also Reference [24]. The results of Cooper, Smith and Vidal [17] are not very different from the Kepple-Griem curves. On each figure is indicated the measured electron density N_e. This value is used for calculating the wavelength distance $|\Delta\lambda_{ep}| = (\lambda_0^2/c)\,\Delta\nu_{ep}$ which corresponds to the electron plasma frequency $\Delta\nu_{ep}$.

All measured profiles show a characteristic structure in the line wings. There appear two lateral peaks which have from the H_β line center ($\Delta\lambda = 0$) a wavelength distance $\Delta\lambda$ corresponding to the electron plasma frequency $\Delta\lambda_{ep}$. More precisely speaking, relative to the H_β line center the electron plasma satellite appear suddenly within one channel width (i. e. within 0.1 Å) and then extend to the more distant line wings were they smooth out. The satellites appeared in any discharge studied during several months and were reproducible. Figure 5 shows

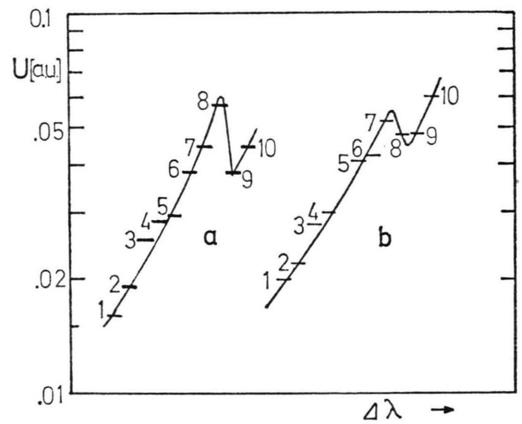

Fig. 5. Output signals of the ten channels for two different shots (for details see text). $N_e = 1 \cdot 10^{15}\ \mathrm{cm}^{-3}$.

the ten output signals for two shots of the same measuring series. The difference between Figs. 5 a and 5 b is that we have displaced the receiving head by a small amount which is a little bit larger than one channel width such that the satellite falls in Fig. 5 b on other channels than in Figure 5 a. As the weak minimum and maximum do not coincide in Fig. 5 b with the center of corresponding channels the satellite is not so pronounced as in Fig. 5 a but still well discernible from the underlying line wing. Further investigation revealed that the two satellites do not lie completely symmetric to the H_β line center, the wavelength distance of the blue satellite being approximately 0.1 Å larger than the one for the red satellite.

We have some experimental indications that a second satellite pair lies at wavelength distances $\pm 2\,\Delta\lambda_{ep}$ from the H_β line center. Due to their very weak intensity it was not possible to separate them with full certainty from the underlying line wing.

The most striking feature is that two profiles (Figs. 2 and 3) show a resolved component in the line center where all quasi-static theories and recent ion dynamic theories predict a deep minimum. A central peak is not seen in the line profile for $N_e = 5 \cdot 10^{14}\,\mathrm{cm}^{-3}$ (Figure 5). It is probably merged in the strong intensity of the nearby lying intensity shoulders. A strong support for this interpretation is the extremely high intensity in the central part of the line which exceeds by more than a factor of two the theoretical intensity for the quasi-static profile. Due to the presence of this unresolved central component the intensity of the whole line core is pushed upward.

As the electron plasma satellites the central component could be resolved with a relatively good precision. Figure 6 shows the output signals of the ten channels when placed in the line core.

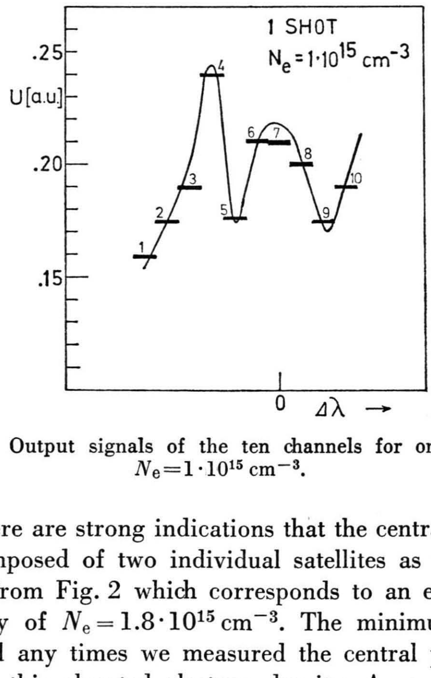

Fig. 6. Output signals of the ten channels for one shot; $N_e = 1 \cdot 10^{15}\,\mathrm{cm}^{-3}$.

There are strong indications that the central peak is composed of two individual satellites as can be seen from Fig. 2 which corresponds to an electron density of $N_e = 1.8 \cdot 10^{15}\,\mathrm{cm}^{-3}$. The minimum appeared any times we measured the central part of H_β at this elevated electron density. As a further proof of this feature may serve Fig. 7 which represents the ensemble of all output signals in the line core measured during several months. For a given channel all measured values fall in the correspond-

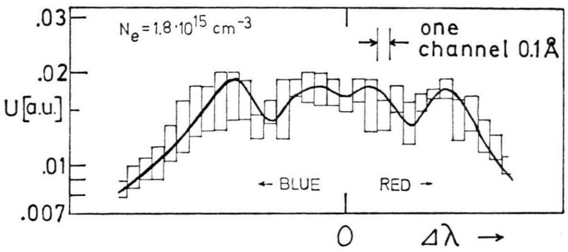

Fig. 7. Ensemble of all channel signals in the line center, measured during the whole measuring campagne of several months; no corrections made.

ing rectangular field. This representation includes possible errors in calibration, variations in filling pressure and condenser voltage and also a change of the discharge tube. Even in this representation the minimum in the line center is still visible. The wavelength distance from the H_β line center of the two weak maxima of the central feature corresponds approximately to three to five times the ion plasma frequency. The ion plasma frequency itself would lead to peaks at $\Delta\lambda_{ip} = \pm 0.067$ Å not discernible with our analyzer system.

The intensity maximum in the line core cannot be caused by plasma inhomogeneity as explained above. Self absorption must also be excluded, the optical depth being to small to cause a significant lowering of the intensity. We think that the central intensity maximum has its origin in collective low-frequency oscillations which couple with the atomic system.

It is possible to estimate the intensity of the plasma satellites relative to the intensity of the whole H_β line. Figure 8 shows the intensity ratio

$$R = \int_{\text{satellite}} I(\Delta\lambda)\,\mathrm{d}(\Delta\lambda) \Big/ \int_{H_\beta} I(\Delta\lambda)\,\mathrm{d}(\Delta\lambda)$$

for the two electron plasma satellites. The intensity in the violet satellite is always approximately two times larger than in the red one.

For the highest electron densities it is possible to estimate the order of magnitude of the intensity contained in the central peak relative to the intensity of the whole H_β line. One finds approximately seven to ten percent. This is five times more than for the electron plasma satellites.

Our measurements revealed further a general asymmetry of the whole H_β line profile, which is larger than generally observed in experiments at high electron densities. An exception seems to be the paper of Wende [25] who found a marked asymmetry

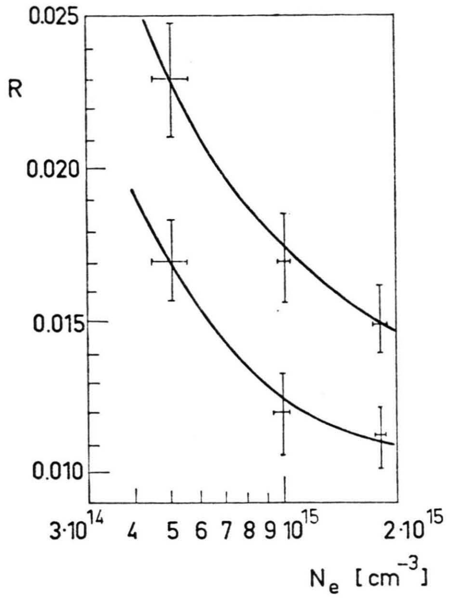

Fig. 8. Intensity in the electron plasma satellites relative to the whole H_β line profile.

of H_β even in the far line wings. The asymmetries found in our experiments are qualitatively the same as those observed by Wende at much higher electron densities ($N_e \cong 7 \cdot 10^{16}\,\mathrm{cm}^{-3}$). The asymmetry effect at low electron densities is surprizing, since it is generally assumed to be due to quadratic Stark effect and higher order multipole contributions. They therefore should play a role only at high electron densities and should be negligible at the densities of our experiment. The question arises whether the observed asymmetry in our experiment is caused by plasma oscillations?

Discussion and Conclusion

Our measurements of the profile of the H_β line emitted by a hydrogen afterglow plasma showed on the one hand weak and reproducible side-band satellites associated with electron plasma waves and on the other hand a reproducible intensity maximum in the line center which is well-separated from the nearby lying H_β shoulders. Neither of these peaks appears in theoretically calculated Stark broadened line profiles. We think that the line component in the line core has its origin in ion waves excited in the plasma, since it cannot be explained by an inhomogeneous plasma layer. When the central component is excited by collective ion oscillations at or near the ion plasma frequency there should appear two side-band components very close to the line center instead of a single maximum. We could not sufficiently resolve the central component but there are experimental indications that it is composed of two individual components as may be seen from Figs. 2 and 7. The probable reasons for the unresolvability of the central component are:

1. Finite resolution of only 0.1 Å per channel of our analyzer system,
2. Thermal Doppler effect of the emitting atoms (full Doppler half-width $2\Delta_D' \cong 0.16$ Å).
3. A rather large spectral width of the ion waves.

Convolution of all these effects can explain the measured intensity distribution. We propose that refined measurements should be made at much lower gas temperatures in order to confirm our suppositions.

The intensity of the central component(s) would not only depend on the ion plasma frequency itself but also on the intensity level of the ion acoustic waves and the coupling strength with the atomic system. The energy contained in these waves will in any case depend on the ion mass. (For infinite ion mass there will be no ion wave.) The mass dependence of the form of the H_β line could thus find its explanation in the existence of such waves.

There arises an important question to which no definite answer can be given at this stage: what is the energy source which excites the electron and ion plasma waves? The electron plasma wave satellites have often been observed in highly turbulent plasmas. In these cases the energy is delivered by energetic electron beams which excite the electron waves. The observed plasma satellites (see e. g. Kunze and Griem [26], Burrell and Kunze [27], Cooper and Ringler [28], C. C. Gallagher and Levine [29], Rutgers and de Kluiver [30], Rutgers and Kalfsbeek [31]) can be explained in the frame of the Baranger and Mozer [32] theory. But electron plasma wave satellites have also been observed in apparently quiescent plasmas with a very low turbulence level (Ya'akobi and Bekefi [33] and Baravian et al. [34]). As these laser-produced plasmas were observed during the recombination phase it is possible that the energy was delivered by supra-thermal electrons produced by superelastic electron collisions (see e. g. Chappel, Cooper and Smith [35]). The same energy source could be responsable for the observed electron plasma satellites in our experiments.

If it is true that collective ion oscillations have such a strong influence on the profiles of spectral lines emitted in the optical region there might be a possibility to use this effect for the study of the spectrum of low-frequency oscillations. However, the applicability will probably be very limited, since one needs low electron densities (otherwise the profile would be dominated by usual Stark effect) and relatively low Doppler temperatures (otherwise the ion wave effect would be masked by Doppler effect). Under special conditions it might however be possible that the ion waves are so strongly excited that a still detectable intensity associated with the ion wave effect lies outside the Doppler core. In any case the electron density must lie above some critical value in order to avoid that the electron plasma satellites do not perturbe the ion wave feature. Diagnostic application will therefore be restricted to very special situations.

Acknowledgements

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[1] H. R. Griem, Astrophys. J. **154**, 111 [1968].
[2] A. J. Barnard, J. Cooper, and L. J. Shamey, Astron. and Astrophysics. **1**, 28 [1969].
[3] D. D. Burgess and C. J. Cairns, J. Phys. B **3**, L67 [1970] and **4**, 1364 [1971].
[4] J. E. Jenkins and D. D. Burgess, J. Phys. B **4**, 1353 [1971].
[5] D. D. Burgess, J. Phys. B **3**, L70 [1970].
[6] H. W. Drawin and J. Ramette, Z. Naturforsch. **29 a**, 838 [1974].
[7] C. S. Diatta, A. Czernichowski, and J. Chapelle, Z. Naturforsch. **30 a**, 900 [1975].
[8] J. R. Greig, L. A. Jones, and R. W. Lee, Phys. Rev. A **9**, 44 [1974].
[9] R. W. Lee, J. Phys. B **5**, **L23**, [1972] and J. Phys. B **6**, 1044 [1973].
[10] A. J. Barnard, J. Cooper, and E. W. Smith, J. Quant. Spectr. Radiative Transf. **14**, 1025 [1974].
[11] A. J. Barnard and D. C. Stevenson, J. Quant. Spect. Radiative Transf. **15**, 123 [1975].
[12] E. R. A. Segré and D. Voslamber, Phys. Lett. **46 A**, 397 [1974], see also E. R. A. Segré, Thesis, Université Paris VII [1975]: "Etude de l'influence de la dynamique des ions d'un plasma sur l'élargissement spectral. Application aux raies 4^3D, $F-2^3P$ de l'helium neutre".
[13] A. Brissaud, C. Goldbach, J. Leorat, A. Mazure and G. Nollez, Research Report 030, July 1975, Ecole Nationale Supérieure de Techniques Avancées, Paris. Article to be published in J. Phys. B.
[14] D. E. Kellerher and W. L. Wiese, Phys. Rev. Lett. **31**, 1431 [1973].
[15] W. L. Wiese, D. E. Kellerher, and V. Helbig, Phys. Rev. A **11**, 1854 [1975].
[16] P. Kepple and H. R. Griem, Phys. Rev. **173**, 317 [1968].
[17] C. R. Vidal, J. Cooper, and E. W. Smith, Astrophys. J. Suppl. **25**, 37 [1973].
[18] D. D. Burgess and R. Mahon, J. Phys. B **5**, 1756 [1972].
[19] R. W. Lee, J. Phys. B **6**, 1060 [1973).
[20] J. Cooper, E. W. Smith, and C. R. Vidal, J. Phys. B **7**, L 101 [1974].
[21] H. Capes, R. Stamm, and D. Voslamber, Paper G. 6. Second Inter. Conf. on Spectral Lines, University of Oregon, Eugene (Oregon), August 26–30 [1974].
[22] R. Stamm and D. Voslamber, CECAM-CEA-Conference on Perspectives for the Theory and Numerical Methods of High Temperature and Pressure Effects in U. V. Spectra, Paris, October 6–10 [1975]; and private communication.
[23] J. D. Hey and H. R. Griem, Phys. Rev. A **12**, 169 [1975].
[24] H. R. Griem, Spectral Line Broadening by Plasma, Academic Press, New York 1974.
[25] B. Wende, Z. Angew. Physik **22**, 181 [1967].
[26] H.-J. Kunze and H. R. Griem, Phys. Rev. Lett. **21**, 1048 [1968].
[27] C. F. Burrell and H.-J. Kunze, Phys. Rev. Lett. **29**, 1445 [1972].
[28] W. S. Cooper and H. Ringler, Phys. Rev. **179**, 226 [1969].
[29] C. C. Gallagher and M. A. Levine, Phys. Rev. Lett. **27**, 1693 [1971]; **30**, 897 [1973].
[30] W. R. Rutgers and H. de Kluiver, Z. Naturforsch. **29 a**, 42 [1974].
[31] W. R. Rutgers and H. W. Kalfsbeek, Z. Naturforsch. **30 a**, 739 [1975].
[32] M. Baranger and B. Mozer, Phys. Rev. **123**, 25 [1961].
[33] B. Ya'akobi and G. Bekefi, Phys. Letters **30 A**, 539 [1969].
[34] G. Baravian, J. Bretagne, J. Godart, and G. Sultan, Z. Phys. B **20**, 255 [1975].
[35] W. R. Chappell, J. Cooper, and E. W. Smith, J. Quant. Spectrosc. Radiative Transf. **10**, 1195 [1970].